

CHEMICAL ENGINEERING

Fluid Mechanics



Comprehensive Theory
with Solved Examples and Practice Questions





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Fluid Mechanics

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Fluid Properties

1.1 INTRODUCTION

- A fluid is a substance which deforms continuously under the influence of shearing forces no matter how small the forces may be.
- Fluids are substance capable of flowing and they conforms to the shape of the containing vessel.
- This property of continuous deformation in technical terms is known as 'flow property', whereas this property is absent in solids.
- If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.
- Fluids are classified as ideal fluids and practical or real fluids.
- Ideal fluids are those fluids which have neither viscosity nor surface tension and they are incompressible. In nature, the ideal fluids do not exist and therefore, they are only imaginary fluids.
- Practical or real fluids are those fluids which possess viscosity, surface tension and compressibility.
- Fluids are considered to be continuum i.e., a continuous distribution of matter with no voids or empty spaces.
- Difference between fluid and solid is that solid can resist a shear stress by static deflection but fluid cannot resist it.

1.2 FLUID MECHANICS

- Fluid mechanics is study of fluids either at rest or in motion.
- Total fluid mechanics can be dealt with two different approaches, empirical hydraulics and classical hydrodynamics.
- Hydraulics is mainly concerned with motion of water. It is based on the physical principles and has close correlation with experimental studies which both complement and substantiate the fundamental analysis.
- Hydrodynamics is essentially mathematical science dealing with flow analysis based on concept of an ideal fluid, a fictitious fluid in which both fluid viscosity and fluid compressibility are assumed absent.

1.3 FLUID AS CONTINUUM

- Since fluids are aggregations of molecules widely spread for gas and closely spaced for a liquid. The distance between molecules is very large compared to molecular diameter.
- The molecules are not fixed in lattice but move about freely. Thus fluid density or mass per unit volume has no practical meaning because the numbers of molecule occupying a given volume continuously changes.
- But if chosen unit volume is too large there could be noticeable variation in the bulk aggregation of particle. So density can be written as

$$\rho = \lim_{\delta v \rightarrow \delta v'} \frac{\delta m}{\delta v}$$

- Since most engineering problems are connected with larger sample volume, so density being a point function and other fluid properties can be thought of as varying continually in space. Such a fluid is called a continuum, which simply means that its variation in properties is so smooth that differential calculus can be used to analyse the substance.

1.4 FLUID PROPERTIES

- Any characteristic of a fluid system is called a fluid property.
- Fluid properties are of two types:
 - (i) Intensive Properties:** Intensive properties are those that are independent of the mass of the fluid system.
Example: Temperature, pressure, density etc.
 - (ii) Extensive Properties:** Extensive properties are those whose values depend on the size or extent of the system.
Example: Total mass, total volume, total momentum etc.
- Following are some of the intensive and extensive properties of a fluid system.
 - (i) Viscosity
 - (ii) Surface tension
 - (iii) Vapour pressure
 - (iv) Compressibility and elasticity

1.4.1 Some other Important Properties

- 1. Mass Density :** Mass density (or specific mass) of a fluid is the mass which it possesses per unit volume. It is denoted by the Greek symbol ρ . In SI system, the unit of ρ is kg/m^3 .
- 2. Specific Gravity :** Specific gravity (S) is the ratio of specific weight (or mass density) of a fluid to the specific weight (or mass density) of a standard fluid. The standard fluid chosen for comparison is pure water at 4°C .

$$\text{Specific gravity of liquid} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of water}} = \frac{\text{Specific weight of liquid}}{9810 \text{ N/m}^3}$$

- 3. Relative Density (R.D.) :** It is defined as ratio of density of one substance with respect to other substance.

$$\rho_{1/2} = \frac{\rho_1}{\rho_2}$$

where, $\rho_{1/2}$ = Relative density of substance '1' with respect to substance '2'.

4. **Specific Weight** : Specific weight (also called weight density) of a fluid is the weight it possesses per unit volume. It is denoted by the Greek symbol γ . For water, it is denoted by γ_w . In SI system, the unit of specific weight is N/m^3 . The mass density and specific weight γ has following relationship $\gamma = \rho g$; $\rho = \gamma / g$. Both mass density and specific weight depend upon temperature and pressure.
5. **Specific Volume** : Specific volume of a fluid is the volume of the fluid per unit mass. Thus it is the reciprocal of density. It is generally denoted by v . In SI unit specific volume is expressed in cubic meter per kilogram, i.e., m^3/kg .

Example 1.1 Three litres of petrol weigh 23.7 N. Calculate the mass density, specific weight, specific volume and specific gravity of petrol.

Solution:

Mass density of petrol,
$$\rho_p = \frac{M}{V} = \frac{W/g}{V} = \frac{W}{gV} = \frac{23.7}{9.8 \times 3} = 0.805 \text{ kg/litre} = 805 \text{ kg/m}^3$$

Mass density of water,
$$\rho_w = 1000 \text{ kg/m}^3$$

Specific gravity of petrol =
$$\frac{805}{1000} = 0.805$$

Specific weight of petrol = Weight per unit volume

$$= \frac{23.7}{3.0} = 7.9 \text{ N/litre} = 7.9 \text{ kN/m}^3$$

Specific volume = volume per unit mass

$$= \frac{1}{\rho_p} = \frac{1}{805} = 1.242 \times 10^{-3} \text{ m}^3/\text{kg}$$

1.4.2 Viscosity

- Viscosity is a property of the fluids by virtue of which they offer resistance to shear or angular deformation.
- It is primarily due to cohesion and molecular momentum exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers.

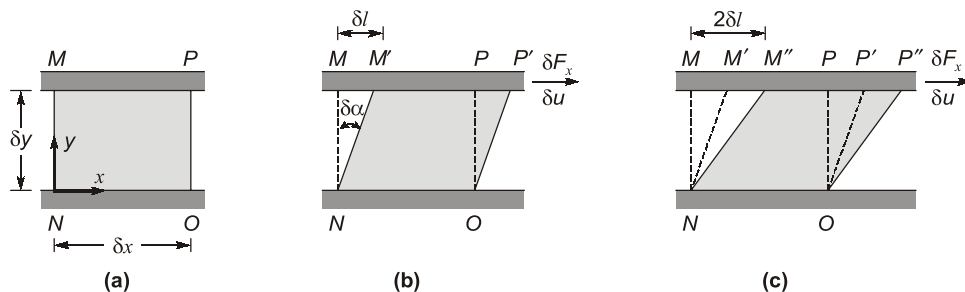


Fig. (a) Fluid element at time t , (b) deformation of fluid element at time $t + \delta t$, and (c) deformation of fluid element at time $t + 2\delta t$.

- Consider the behavior of a fluid element between the two infinite plates as shown in Fig. (a). The rectangular fluid element is initially at rest at time t . Let us now suppose a constant rightward force δF_x is applied to the upper plate so that it is dragged across the fluid at constant velocity δu . The relative shearing action of the plates produces a shear stress, τ_{yx} , which acts on the fluid element and

is given by $\tau_{yx} = \lim_{\delta A_y \rightarrow 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$, where δA_y is the area of contact of the fluid element with the plate and δF_x is the force exerted by the plate on that element.

Various positions of the fluid element, shown in Fig. illustrate the deformation of the fluid element from position $MNOP$ at time t , to $M'NOP'$ at time $t + \delta t$, to $M''NOP''$ at time $t + 2\delta t$, due to the imposed shear stress. The deformation of the fluid is given by

$$\text{Deformation rate} = \lim_{\delta t \rightarrow 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$$

Distance between the points M and M' is given by,

$$\delta l = \delta u \delta t \quad \dots(i)$$

Alternatively, for small angles,

$$\delta l = \delta y \delta \alpha \quad \dots(ii)$$

Equating Eq. (i) and (ii), we get, $\delta u \delta t = \delta y \delta \alpha$

$$\text{or} \quad \frac{\delta \alpha}{\delta t} = \frac{\delta u}{\delta y}$$

Taking the limits of both sides

$$\lim_{\delta t \rightarrow 0} \frac{\delta \alpha}{\delta t} = \lim_{\delta t \rightarrow 0} \frac{\delta u}{\delta y}$$

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$

Thus, the rate of angular deformation is equal to velocity gradient across the flow.

- On the basis of relation between the applied shear stresses and the flow or rate of deformation, fluids can be categorized as Newtonian and non-Newtonian fluids.

1.4.2.1 Newtonian Fluids

- Fluids which obey Newton's law of viscosity are known as Newtonian fluids.
- According to Newton's law of viscosity, shear stress is directly proportional to the rate of deformation or velocity gradient across the flow.

$$\text{Thus,} \quad \tau \propto \frac{du}{dy} \quad \text{or} \quad \tau = \mu \frac{du}{dy}$$

where, $\mu =$ absolute or dynamic viscosity

- Water, air, and gasoline are Newtonian fluids under normal conditions.

Dynamic Viscosity (μ)

- Dimension of $\mu = [M L^{-1} T^{-1}]$
- Unit of $\mu = \text{N-s/m}^2$ or Pa.s
- In c. g. s. units, μ is expressed as 'poise', 1 poise = 0.1 N-s/m²
- (μ) water $\approx 10^{-3}$ N-s/m²;
- (μ) air $\approx 1.81 \times 10^{-5}$ N-s/m² (Both at 20°C and at standard atmospheric pressure)

NOTE: Water is nearly 55 times viscous than air.

Kinematic Viscosity (ν)

- The kinematic viscosity (ν) is defined as the ratio of dynamic viscosity to mass density of the fluid therefore, $\nu = \mu/\rho$

- It is called kinematic because the mass unit cancel, leaving only the dimensions.
- Dimension of $\nu = [L^2 T^{-1}]$
- Unit of $\nu = m^2/s$ or cm^2/s (stoke)
- 1 stoke = $10^{-4} m^2/s$
- At 20°C and atmospheric pressure $\nu_{water} = 1.0 \times 10^{-6} m^2/s$, $\nu_{air} = 15 \times 10^{-6} m^2/s$

NOTE: Kinematic viscosity of air is about 15 times greater than the corresponding value of water.

Effect of Temperature on Viscosity

- It is necessary to understand the factors contributing to viscosity to analyse temperature effect.
- In liquid, viscosity is caused by intermolecular attraction force which weaken as temperature rises so viscosity decreases.
- In gases, viscosity is caused by the random motion of particle/ molecules. Due to increase in temperature, randomness increases causing increase in viscosity.
- For liquids viscosity decreases with temperature and it is roughly exponential as

$$\mu = ae^{-bT}$$

where a and b are constant for a particular liquid.

For water $a = -1.94$, $b = -4.80$

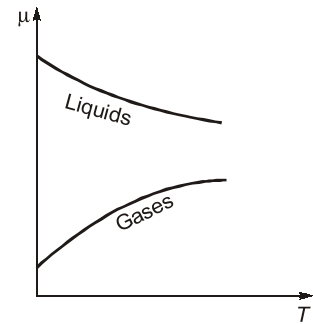


Fig. Variation of Viscosity with Temperature

1.4.2.2 Non-Newtonian Fluids

- Fluids for which shear stress is not directly proportional to deformation rate are non-Newtonian. Toothpaste and paint are the examples of non-Newtonian fluid.
- Non-Newtonian fluids commonly are classified as having time-independent or time-dependent behavior.

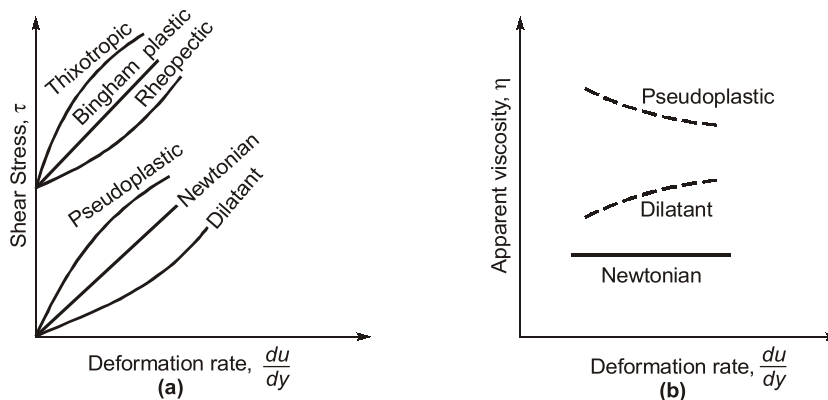


Fig. (a) Shear stress, τ and (b) Apparent viscosity, η

- Relation between shear stress and rate of deformation for non-Newtonian fluid can be represented as

$$\tau = k \left(\frac{du}{dy} \right)^n$$

where, n = flow behavior index; k = consistency index

For Newtonian fluid, $n = 1$; $k = \mu$

above equation can also be represented as

$$\tau = k \left(\frac{du}{dy} \right)^{n-1} \left(\frac{du}{dy} \right) = \eta \frac{du}{dy}$$

where, $\eta = k \left(\frac{du}{dy} \right)^{n-1}$ is referred as the apparent viscosity

NOTE: Dynamic viscosity (μ) is constant (except for temperature effects) while apparent viscosity (η) depends on the shear rate.

- Various types of non-Newtonian fluids are :
 1. **Pseudoplastic** : Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$) are called pseudoplastic fluids (or shear thinning). Most non-Newtonian fluids fall into this group.
Example: Polymer solutions, colloidal suspensions, milk, blood and paper pulp in water.
 2. **Dilatant** : If the apparent viscosity increases with increasing deformation rate ($n > 1$), the fluid is termed as dilatant (or shear thickening).
Example: Suspensions of starch, saturated sugar solution.
 3. **Bingham Plastic** : Fluids that behave as a solid until a minimum yield stress, τ_y , and flow after crossing this limit are known as ideal plastic or Bingham plastic. The corresponding shear stress model is $\tau = \tau_y + \mu \frac{du}{dy}$.
Example: Clay suspensions, drilling muds, creams and toothpaste.
 4. **Thixotropic** : Apparent viscosity (η) for thixotropic fluids decreases with time under a constant applied shear stress.
Example: Paints, printer inks
 5. **Rheopectic** : Apparent viscosity (η) for rheopectic fluids increases with time under constant shear stress.
Example: Gypsum pastes.

NOTE

- (i) There is no relative movement between fluid attached to the solid boundary and solid boundary i.e. the fluid layer just adjacent to the solid surface will have same velocity as of the solid surface.
- (ii) Viscoelastic : Fluids which after some deformation partially return to their original shape when the applied stress is released such fluids are called viscoelastic.
- (iii) Rheology : Branch of science which deals with the studies of different types of fluid behaviours.

Example 1.2

Calculate the velocity gradient at distance 0, 100, 150 mm from the boundary if the velocity is a parabola with vortex 150 mm from boundary, where velocity is 1 m/s. Also calculate the shear stress at these points if the fluids has a viscosity of 0.804 Ns/m².

Solution:

Let the equation of velocity profile

$$u = Ay^2 + By + C$$

Now apply boundary condition

(i) $u = 0$ at $y = 0 \Rightarrow c = 0$

(ii) $u = 1 \text{ m/s}$ at $y = 0.15 \text{ m}$
 $1 = 0.15^2 \times A + 0.15 B \quad \dots(\text{ii})$

(iii) at $y = 0.15 \text{ m}$ at $\frac{du}{dy} = 0$

$$\frac{du}{dy} = 2Ay + B$$

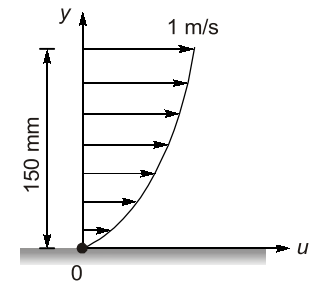
$$2A \times 0.15 + B = 0 \quad \dots(\text{iii})$$

From Eq. (ii) and (iii), we get,

$$A = -44.4; \quad B = 13.33$$

So velocity profile will be given as

$$u = -44.4 y^2 + 13.33 y$$



(a) at $y = 0 \text{ mm}$ $\frac{du}{dy} = -2 \times 44.4 \times 0 + 13.33 = 13.33 \text{ sec}^{-1}$

Shear stress, $\tau = \mu \frac{du}{dy} = 0.804 \times 13.33 = 10.8 \text{ N/m}^2$

(b) at $y = 100 \text{ mm}$ $\frac{du}{dy} = -2 \times 44.4 \times 0.1 + 13.33 = 4.45 \text{ sec}^{-1}$

$$\tau = \mu \frac{du}{dy} = 0.804 \times 4.45 = 3.575 \text{ N/m}^2$$

(c) at $y = 150 \text{ mm}$ $\frac{du}{dy} = -2 \times 44.4 \times 0.15 + 13.33 = 0$

$$\tau = 0$$

1.4.3 Surface Tension

- It is a force which exists on the surface of a liquid when it is in contact with another fluid or a solid boundary. Its magnitude depends upon the relative magnitude of cohesive and adhesive forces.
- Surface tension is a force in the liquid surface and acts normal to a line of unit length drawn imaginarily on the surface. Thus it is a line force.
- It represents surface energy per unit area. It has dimension MT^{-2} and SI unit is N/m .
- Whenever a liquid is in contact with other liquids or gases the interface develops that acts like a stretched elastic membrane, creating surface tension.

Example 1.3

A circular disc of diameter d is slowly rotated in a liquid of large viscosity ' μ ' at a small distance ' h ' from fixed surface. Derive expression for torque ' T ' necessary to maintain an angular velocity ' ω '.

Solution:

Consider an element of disc at radius r and having a width dr

Linear velocity at this radius

$$V = r\omega$$

$$\text{Torque} = \text{Shear stress} \times \text{Area} \times r$$

$$\text{Shear stress, } \tau = \frac{\mu du}{dy}$$

Torque required for small ring, dT

$$\therefore dT = \frac{\mu du}{dy} \times 2\pi r \cdot dr \cdot r$$

Now assuming that h is very small and velocity distribution is linear. So,

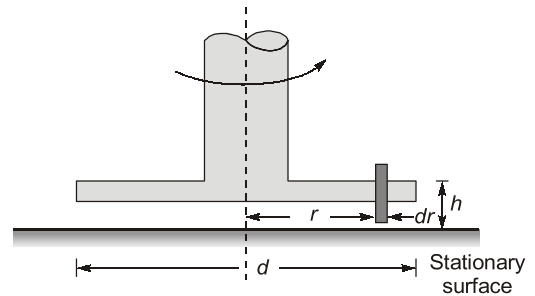
$$\frac{du}{dy} = \frac{r\omega}{h}$$

$$\therefore dT = \frac{\mu r \omega}{h} \times 2\pi r^2 dr = \frac{2\pi\mu\omega}{h} r^3 dr$$

So total torque required,

$$T = \int_0^{d/2} dT = \frac{2\pi\mu\omega}{h} \left[\frac{r^4}{4} \right]_0^{d/2}$$

$$T = \frac{\mu\pi d^4 \omega}{32h}$$



Effect of Temperature

- As the surface tension depends directly upon the intermolecular cohesion and since this cohesion is known to decrease with temperature rise the surface tension decreases with rise of temperature.
- Its value for water-air contact (free-surface of water) reduces from 0.0731 N/m at 17.8°C to 0.0585 N/m at 100°C.

Droplet and Jet

- When a droplet is separated initially from the surface of the main body of liquid, then due to surface tension there is a net inward force exerted over the entire surface of the droplet which causes the surface of the droplet to contract from all the sides and results in increasing the internal pressure within the droplet.
- The contraction of the droplet continues till the inward force due to surface tension is in balance with the internal pressure and the droplet forms into sphere which is the shape for minimum surface area.
- The internal pressure within a jet of liquid is also increased due to surface tension.
- The internal pressure intensity within a droplet and a jet of liquid in excess of the outside pressure intensity may be determined by the expressions derived below.

- (i) **Pressure intensity inside a droplet** : Consider a spherical droplet (Fig. (a)) of radius r having internal pressure intensity p in excess of the outside pressure intensity. If the droplet is cut into two halves, then the forces acting on one half (Fig. (b)) will be those due to pressure intensity (p) on the

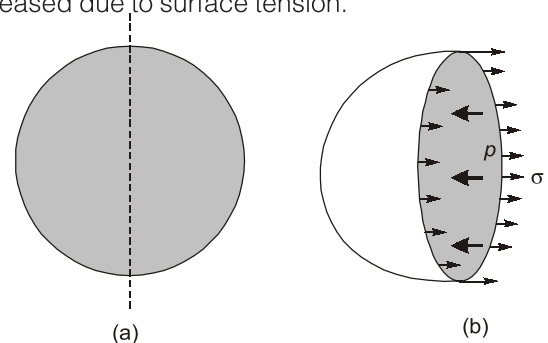


Fig. Surface Tension (σ) and Internal Pressure (p) in a droplet

projected area (πr^2) and the tensile force due to surface tension (σ) acting around the circumference ($2\pi r$). These two forces will be equal and opposite for equilibrium and hence we have

$$\rho(\pi r^2) = \sigma(2\pi r)$$

or
$$\rho = \frac{2\sigma}{r}$$

NOTE: Above equation indicates that the internal pressure intensity increase with the decrease in the size of droplet.

(ii) **Pressure intensity inside a soap bubble :** A spherical soap bubble has two surfaces in contact with air, one inside and the other outside, each one of which contributes the same amount of tensile force due to surface tension (Fig.). As such on a hemispherical section of a soap bubble of radius r , the tensile force due to surface tension is equal to $2\sigma(2\pi r)$. However, the pressure force acting on the hemispherical section of the soap bubble is same as in the case of a droplet and it is equal to $\rho(\pi r^2)$. Thus equating these two forces for equilibrium, we have

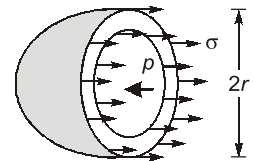


Fig. Soap Bubble

$$\rho(\pi r^2) = 2\sigma(2\pi r) \quad \text{or} \quad \rho = \frac{4\sigma}{r}$$

(iii) **Pressure intensity inside a liquid jet :** Consider a jet of liquid of radius r , length l and having internal pressure intensity p in excess of outside pressure intensity. If the jet is cut into two halves, then the forces acting on one half will be those due to pressure intensity (p) on the projected area ($2rl$) and the tensile force due to surface tension (σ) acting along the two sides ($2l$). These two forces will be equal and opposite for equilibrium and hence we have (Fig.),

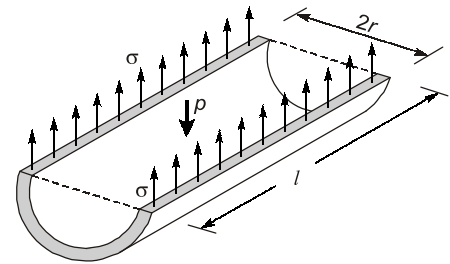


Fig. Liquid Jet

$$\rho(2rl) = \sigma(2l)$$

or
$$\rho = \frac{\sigma}{r}$$

Example 1.4

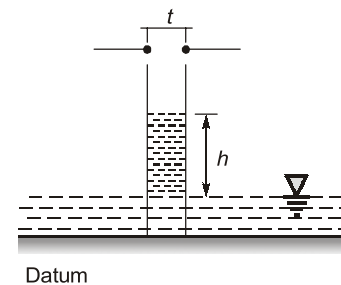
Define capillarity. Derive an equation for capillarity rise between two thin vertical plates spaced 't' distance apart. Calculate the distance between the plates when the capillarity rise is not to exceed 60 mm. Assume surface tension of water at 20°C as 0.075 N/m.

Solution:

Capillarity "It is the phenomenon of rise or fall of liquid surface related to adjacent general level of liquid, due to surface tension, when it is passing through tubes of small thickness."

Let width of plate be L and contact angle be θ so for two vertical plates 't' distance apart. Force due to surface tension = Force due to gravity.

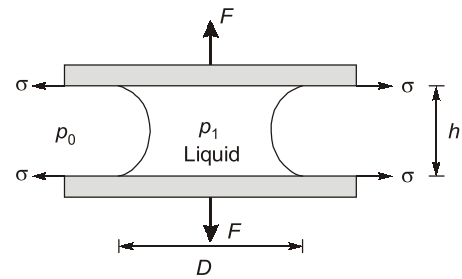
$$\sigma \times (2L) \cos \theta = \rho g(L \times t) \times h$$



$$\Delta p = \frac{2\sigma}{h}$$

So force required to pull the plate apart

$$\begin{aligned} F &= \left(\frac{\pi D^2}{4} \right) (\Delta p) \\ &= \frac{2\pi D^2 \sigma}{4h} = \frac{\pi}{2} \left(\frac{D}{h} \right) \sigma D \end{aligned}$$



Summary



- A fluid is a substance that deforms continuously when subjected to even an infinitesimal shear stress.
- Fluid mechanics is the study of fluids at rest or in motion.
- The concept of a continuum assumes a continuous distribution of mass within the matter or system with no empty space.
- Viscosity is the property of a fluid by virtue of which it offers resistance to flow.
- The rate of deformation of any fluid element in a fluid flow is equal to the velocity gradient across the flow.
- Fluids which obey Newton's law of viscosity are called Newtonian fluids and which do not obey are called non-Newtonian fluids.
- It is due to surface tension that a curved liquid interface, in equilibrium, results in a greater pressure at concave side than that at its convex side.
- A liquid wets a solid surface and results in a capillary rise when the forces of cohesion between the liquid molecules are lower than the forces of adhesion between the molecules of the liquid and the solid in contact.
- Cavitation occurs when the local pressure reduces below vapour pressure of the liquid.
- Compressibility of a substance is the measure of its change in volume or density under the action of external forces.



Important Expressions

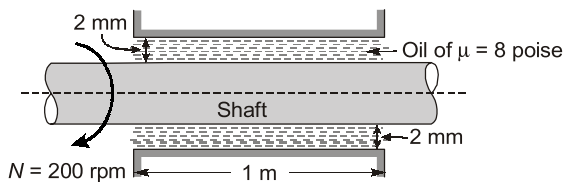
- Deformation rate $\left(\frac{d\alpha}{dt} \right) = \text{velocity gradient} \left(\frac{du}{dy} \right)$
- Shear stress for Newtonian fluids : $\tau = \mu \left(\frac{du}{dy} \right)$
- Shear stress for non-Newtonian fluids: $\tau = k \left(\frac{du}{dy} \right)^n = k \left(\frac{du}{dy} \right)^{n-1} \frac{du}{dy} = \eta \frac{du}{dy}$
- Shear stress for Bingham plastics : $\tau = \tau_y + \mu \frac{du}{dy}$
- Pressure intensity p in excess of the outside pressure intensity in a droplet : $p = \frac{2\sigma}{r}$

- Pressure intensity p in excess of the outside pressure intensity in a soap bubble: $p = \frac{4\sigma}{r}$
- Pressure intensity p in excess of the outside pressure intensity in a liquid jet : $p = \frac{\sigma}{r}$
- Capillary rise or fall : $h = \frac{2\sigma \cos\theta}{S\gamma_w r}$
- Compressibility : $\beta = \frac{1}{K} = \frac{-(\Delta V/V)}{dp} = \frac{(dp/\rho)}{dp}$



Objective Brain Teasers

- Q.1** If the dynamic viscosity of a liquid is 0.012 poise and its R.D. is 0.79, then its kinematic viscosity in stoke is
 (a) 0.0152 (b) 0.152
 (c) 1.52 (d) 15.20
- Q.2** The velocity distribution, in m/s near the solid wall at a section in a laminar flow is given by $u = 5 \sin(5\pi y)$. If $\mu = 5$ poise, the shear stress at $y = 0.05\text{m}$, in N/m^2 is
 (a) 39.27 (b) 27.77
 (c) 38.9 (d) 26.66
- Q.3** A solid shaft diameter of 350 mm, rotates at 200 rpm inside a fixed sleeve bearing as shown in figure. The dynamic viscosity of oil is 8 poise.



The power lost (in kW) due to viscosity in bearing is

- (a) 4.9 (b) 5.9
 (c) 11.8 (d) 2.95
- Q.4** A fluid indicated the following shear stress and deformation rates :

$\frac{du}{dy}$ (units)	0	1	2	4
τ (units)	10	15	20	30

This fluid is classified as

- (a) Newtonian (b) Bingham Plastic
 (c) Dilatant (d) Pseudoplastic
- Q.5** Kerosene is known to have a bulk modulus of elasticity $K = 1.43 \times 10^9 \text{ N/m}^2$ and a relative density of 0.806. The speed of sound in kerosene, (in m/s) is
 (a) 1332 (b) 1075
 (c) 1197 (d) 184
- Q.6** If 5.66 m^3 of oil weighs 4765 kg, then its mass density, specific weight and specific gravity respectively are
 (a) 841.87 kg/m^3 , 8.26 kN/m^3 and 0.842
 (b) 8.26 kg/m^3 , 841 kN/m^3 and 8.42
 (c) 841.87 kg/m^3 , 841 kN/m^3 and 8.42
 (d) None of these

- Q.7** A reservoir of capacity 0.01 m^3 is completely filled with a fluid of coefficient of compressibility $0.75 \times 10^{-9} \text{ m}^2/\text{N}$. The amount of fluid that spill over (in m^3), if pressure in the reservoir is reduced by $2 \times 10^7 \text{ N/m}^2$ is
 (a) 0.15×10^{-4} (b) 1×10^{-4}
 (c) 1.5×10^{-4} (d) None of these

- Q.8** Assuming that sap in trees has the same characteristic as water and that it rises purely due to capillary phenomenon, what will be the average diameter of capillary tubes in a tree if the sap is carried to a height of 10m? (Take surface tension of water = 0.0735 N/m and $\theta = 0^\circ$)

- (a) 0.003 mm (b) 0.03 mm
(c) 0.3 mm (d) 0.006 mm

Q.9 A small circular jet of mercury 0.1 mm in diameter issue from an opening. What is the pressure difference between the inside and outside of the jet when at 20°C? (Surface tension of mercury at 20°C is 0.514 N/m)

- (a) 41 kPa (b) 21.5 kPa
(c) 10.28 kPa (d) 5.14 kPa

Q.10 An apparatus produces water droplets of diameter 70 μm. If the coefficient of surface tension of water in air is 0.07 N/m, the excess pressure in these droplets, in kPa, is

- (a) 5.6 (b) 4.0
(c) 8.0 (d) 13.2

Q.11 If the surface tension of water air interface is 0.073 N/m, the gauge pressure inside a rain drop of 1 mm diameter is

- (a) 146.0 N/m² (b) 0.146 N/m²
(c) 73.0 N/m² (d) 292.0 N/m²

Q.12 The capillary rise in a 3 mm tube immersed in a liquid is 15 mm. If another tube of diameter 4 mm is immersed in the same liquid, the capillary rise would be

- (a) 11.25 mm (b) 20.00 mm
(c) 8.44 mm (d) 26.67 mm

Q.13 Which of the following is the correct expression for the bulk modulus of elasticity of a fluid?

- (a) $\rho \frac{dp}{dp}$ (b) $\rho \frac{dp}{dp}$
(c) $\frac{dp}{\rho dp}$ (d) $\frac{dp}{\rho dp}$

Q.14 A Newtonian fluid fills the clearance between a shaft and a sleeve. When a force of 800 N is applied to the shaft, parallel to the sleeve, the shaft attains a speed of 1.5 cm/s. If a force of 2.4 kN is applied instead, the shaft would move with a speed of

- (a) 1.5 cm/s (b) 13.5 cm/s
(c) 0.5 cm/s (d) 4.5 cm/s

Q.15 If the shear stress τ and shear rate $\left(\frac{du}{dy}\right)$

relationship of a material is plotted with τ on

the Y-axis and $\frac{du}{dy}$ on the X-axis, the behaviour

of an ideal fluid is exhibited by

- (a) a straight line passing through the origin and inclined to the X-axis
(b) the positive X-axis
(c) the positive Y-axis
(d) a curved line passing through the origin

ANSWERS

- 1.** (a) **2.** (b) **3.** (b) **4.** (b) **5.** (a)
6. (a) **7.** (c) **8.** (a) **9.** (c) **10.** (b)
11. (d) **12.** (a) **13.** (b) **14.** (d) **15.** (b)



Student's Assignments

Q.1 A glass tube 0.25 mm in diameter contains a mercury column with water above the mercury. The temperature is 20°C at which the surface tension of mercury in contact with water is 0.037 kg(f)/m. What will be the capillary depression of the mercury? Take angle of contact $\theta = 130^\circ$.

Ans. 3.02 cm

Q.2 The velocity distribution in the flow of a thin film of oil down an inclined channel is given

$$\text{by } u = \frac{\gamma}{2\mu}(d^2 - y^2)\sin\alpha, \text{ where, } d = \text{depth}$$

of flow, $\alpha =$ angle of inclination of the channel to the horizontal, $u =$ velocity at a depth y below the free surface, $\gamma =$ unit weight of oil and $\mu =$ dynamic viscosity of oil. Calculate the shear stress : (i) on the bottom of the channel, (ii) at mid - depth, and (iii) at the free surface.